Barnett *et al.* Reply: Torcini *et al.* [1] raise some interesting issues. Their main point is that the diffusion coefficient of a dilute gas diverges with decreasing density, while the Lyapunov exponent tends to zero; surely they cannot be related by a $\frac{1}{3}$ power rule.

However, Torcini *et al.* overlook elementary dimensional analysis, which shows that the proportionality constant in the relationship would have to be density dependent. Equivalently, the Lyapunov exponent and diffusion constant must be rescaled by system parameters into dimensionless quantities before being compared by the $\frac{1}{3}$ power rule. This they have not done. More specifically, the expression $\tilde{\lambda}_1 \propto \tilde{D}^{1/3}$ is obtained in the normalization of $\tilde{\lambda}_1 = \frac{\lambda_1}{\omega_p}$ and $\tilde{D} = D/(ma^2\omega_p)$, where $\omega_p = (4\pi ne^2/m)^{1/2}$ and $a = [3/(4\pi n)]^{1/3}$. However, the normalization of the simulation results for a hand sphere gas in Fig. 1 [1] is not that of our expression.

Unfortunately, for a dilute hard sphere gas, the hard sphere radius and number density can be combined into a dimensionless quantity by themselves. Consequently, any arbitrary relationship between the Lyapunov exponent and diffusion can be matched in the very dilute regime when density is the control parameter, which is not very useful.

In addition to their main point, Torcini *et al.* allege that our example is outside the scope of our theory because it is a dense plasma, and the Coulomb force is "long range." However, the theory requires diluteness only in the sense that three-body and four-body interactions contribute negligibly to the autocorrelation integrals for the second derivative of the potential (i.e., binary collisions). This is met even for liquid plasmas. Indeed, with dense plasmas, the Debye length is shorter than the inter-ion spacing so the effective interaction is short range. Even in a sparse plasma, the range of the second derivative of the Coulomb potential goes as $(1/r)^3$ which is still "short range."

Our *ab initio* theory yields a fundamental result equating the Lyapunov exponent to a function of integrals of autocorrelations for fluctuations in the second derivative of the potential. The "diluteness" (in the sense of binary collisions) and equilibrium simplify the function to the $\frac{1}{3}$ power of an autocorrelation integral, c_0 [Eq. (26)]. The expressions clearly yield the correct limiting behavior of a Lyapunov exponent which tends to zero with the density.

We observed that the fluctuation-dissipation theorem (or Kubo formulas) [2] relates transport coefficients to time integrals of fluctuation autocorrelations for corresponding dynamical variables—a general result. This led to our suggestion that the Lyapunov exponent would be proportional to a positive power of the transport coefficients.

We used self-diffusion as our example and had data (now published by Ueshima *et al.* [3]) for a relatively dense one-component plasma. c_1 and c_2 bear a simple relation to c_0 such that the solutions of the secular equation still scale as $c_0^{1/3}$. Barnett and Tajima [4] applied the *ab initio* theory in detail to a one-component plasma.

The real substance of Torcini *et al.*'s criticism is that the second derivative autocorrelation does not seem to parallel the velocity autocorrelation for a dilute gas. Why might this be?

The velocity autocorrelation integral in a gas is dominated by the large times between collisions—indicative of the increasingly ballistic nature of the particle motions. The natural length scale is a mean free path which goes as inverse density. In the limit of ballistic motion, there are no collisions and no fluctuations.

By contrast, in a liquid, a molecule spends time rattling within a cage made by its neighbors. Diffusion takes place in hops from cage to cage, mediated by collisions. The hop represents a significant mean velocity fluctuation. Similarly, the second derivative autocorrelation is dominated by the duration of the collisions themselves (i.e., a dissipative process). In the liquid regime, the cage size, which goes as the $\frac{1}{3}$ power of the inverse density, provides the natural length scale.

Transport, in general, has a coherent (e.g., ballistic) part and a dissipative part. The Lyapunov exponent parallels dissipative transport only. Hence, our original proposal needs to be read with this caution.

Our Lyapunov theory cannot be applied naively to hard spheres because the hard sphere potential is not differentiable. But the effort should be made because of the historical importance of this abstraction.

A $\frac{1}{3}$ power relation between short time exponential expansion and long term diffusion was found by Seki *et al.* [5], and Dupree [6] found, for plasma turbulence, that a mode's growth rate was proportional to the $\frac{1}{3}$ power of the velocity diffusion coefficient. It may be a feature of systems where particles behave singly as if they are scattering off a chaotic background.

Our method clearly makes a very general connection between fluctuations and the largest Lyapunov exponent. Equally clearly, we are far from understanding fully the fascinating connection between Lyapunov expansion in phase space and dissipative transport processes.

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