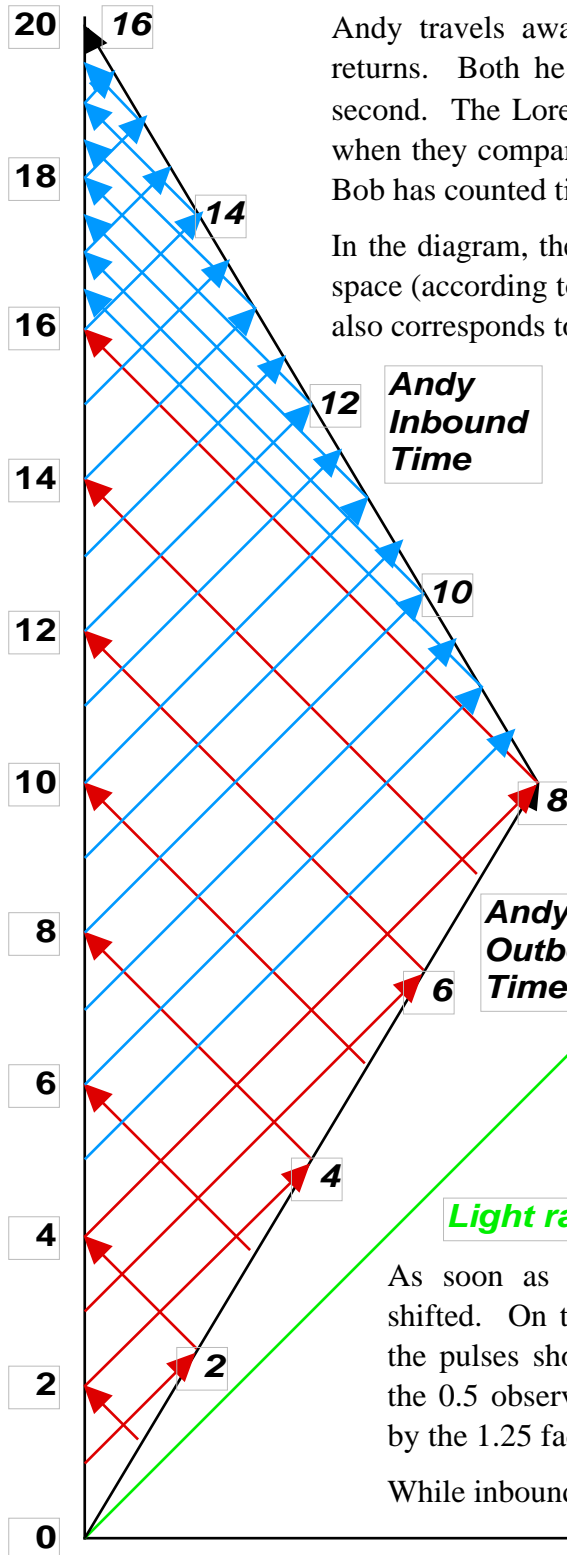


Understanding the Symmetry and Asymmetry of the Special Relativity “Twin Paradox”



Andy travels away from home at $v=0.6c$ for 8 seconds then returns. Both he and his brother, Bob, send out pulses every second. The Lorentz factor is $(1-(v/c)^2)^{-1/2} = 1.25$. At the end, when they compare clocks (and counts of pulses), they find that Bob has counted time 25% faster than Andy.

In the diagram, the vertical axis is time and the horizontal axis is space (according to Bob). Andy's path is shown at an angle. (and also corresponds to his personal time-axis).

The light pulses are shown as 45° lines with an arrow indicating the direction of the recipient and the colour (red or blue) indicates the Doppler shift he perceives.

For the first half of Andy's journey, they each receive a pulse every 2 seconds (i.e. one pulse from his brother for every 2 he sends). On the assumption that each thinks he is stationary with respect to the "ether", they would calculate the classical Doppler shift according to a moving source—the pulses should be arriving every 1.6 seconds not 2—and conclude the other's clock slowed by 1.25.

As an aside: had they assumed a stationary source and moving observer, they would have a relative velocity for the light of $0.4c$ and expect the pulse to arrive every 2.5 seconds and so conclude the other's clock must be **fast** by that 1.25 factor.

During the outbound journey, Andy sends out 8 pulses and receives 4 from Bob.

As soon as Andy turns round, his pulses become blue shifted. On the moving source theory, each concludes that the pulses should be arriving every 0.4 seconds (instead of the 0.5 observed) so concludes that his brother is still slow by the 1.25 factor.

While inbound, Andy sends 8 pulses and receives 16.

Bob receives 8 pulses during the first 16 seconds that Andy is away (sent while Andy was outbound) and himself sent out 16 pulses. During the next 4 seconds he receives 8 pulses (the increase in frequency indicating that Andy sent them while inbound). Bob, himself, has time to send 4.

While in any one leg of Andy's journey, the situation is entirely symmetrical, the crucial difference between Bob and Andy is that Andy does not stay in a single inertial frame, but switches half-way from outbound to inbound.